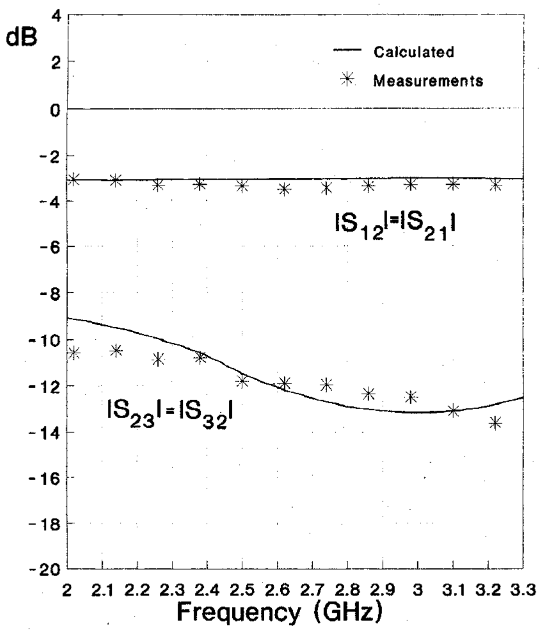

Fig. 5. Transfinite-element and experimental measurements for $|S_{12}|$, $|S_{21}|$, $|S_{23}|$, and $|S_{32}|$ of an equal-split power divider.

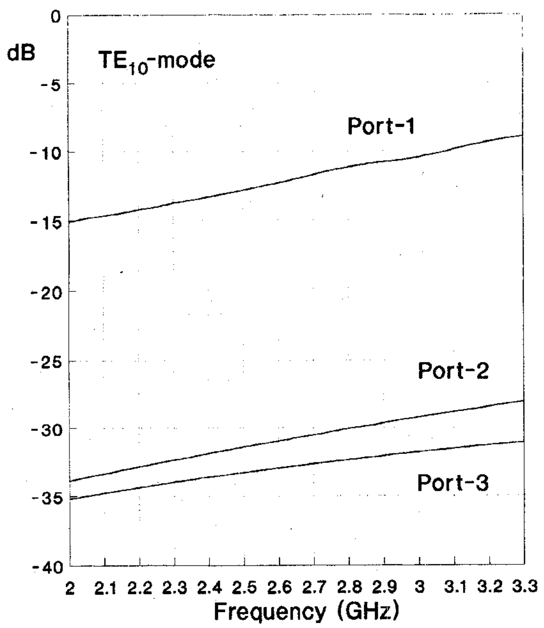

Fig. 6. Calculated amplitude of the TE_{10} mode at each port of an equal-split power divider for an excitation wave at port-2.

ACKNOWLEDGMENT

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An Efficient Synthesis Technique of Tapered Transmission Line with Loss and Dispersion

Eui Joon Park

***Abstract*—A sythesis technique of lossy and dispersive tapered transmission line is newly presented that extends lossless cases suggested by Klopfenstein [1] and others [2]–[4]. A special optimization process based on the Fourier transform pair [5] and generalized Taylor's procedure [6] is performed to extract the exact null points of lobe-like frequency response in terms of the input reflection coefficient of lossy-tapered line in which the loss may be frequency dependent and distance dependent. The theory is verified by evaluation of a synthesized microstrip taper profile in the lossy case and is expected to be helpful for design of tapered line in the high-frequency microwave integrated circuits (MIC's) with loss.**

I. INTRODUCTION

The tapered transnsmission line has been widely used in monolithic microwave/millimeter-wave integrated circuits (MMIC's) and high clock rate digital integrated circuits for the impedance transformation.

So far, no accurate synthesis for a specified frequency response representing input reflection coefficient has been given for lossy tapered transmission lines. Consider a tapered line, supporting a non-TEM mode, which is used as a transformer to match a line of impedance to a load of impedance (Fig. 1). It is well known that the input reflection coefficient for a tapered line is expressible to good approximated solution (for the case of $\rho \ll 1$) of the Riccati equation [1], [5]

$$\rho_i(\omega) = \int_0^L \frac{d \ln \overline{Z}(\omega, z)}{dz} \cdot \exp\left[-2\int_0^z \alpha(\omega, z')\, dz'\right]$$

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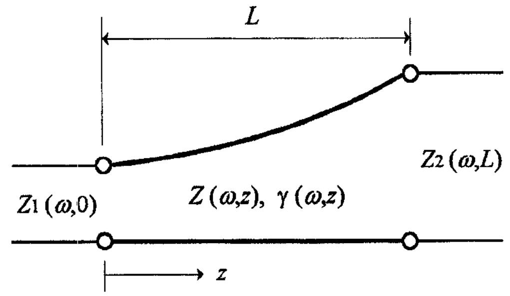

Fig. 1. Tapered transmission line.

$$\cdot \exp\left[-2\int_0^z \beta(\omega, z')\, dz'\right] dz. \tag{1}$$

Here $\overline{Z} = Z/Z_1$ is a normalized impedance that is a function of frequency and distance z along the taper and β is a phase constant. In previous synthesis [2], [4] for the normal cases such as exponential or Chebyshev response, the attenuation constant $\alpha(\omega, z)$ has not been considered. If the characteristic impedance $Z(\omega, z)$ is required to satisfy an arbitrary specified lobe heights of ρ_i in the frequency domain, conventional Fourier transform pair [5] is insufficient because the taper profile for other than the normal cases is previously not known. If the taper profile is achieved in terms of a desired frequency response in the lossless state, the line will have a deviation of the response by loss. So the deviation makes the synthesized lossless tapered line have an arbitrary response.

The concept of control of lobe height was originally proposed by Hyneman [6] who has employed a deterministic perturbation of the zeros of the antenna pattern function to suppress sidelobe levels in particular prescribed angular directions. This method has been applied to the design of tapered lines with a controlled frequency response by Mahon and Elliott [3]. However, this method is limited to lossless lines with fixed phase constant.

In this paper, another synthesis method is proposed introducing conventional Fourier transform pair and generalized Taylor's procedure in cases where frequency-dependent loss and dispersive property, which are functions of distance along the tapered line, are considered. The synthesis technique by proper optimization process to control null points in the lossy case is applied to the design of a microstrip impedance transformer. The result shows that taking the effect of loss on the determination of taper profile into consideration is expected to be helpful especially for high-frequency dispersive integrated circuits and interconnects with high loss.

II. Synthesis

In the lossless case ($\alpha = 0$), (1) is expressible by the following Fourier transform pair [5]

$$f(u) = \int_{-\pi}^{\pi} g(p) e^{-jpu}\, dp \tag{2}$$

$$g(p) = \frac{1}{2\pi}\int_{-\infty}^{\infty} f(u) e^{jpu}\, du = \frac{1}{2}\frac{d\ln \overline{Z}}{dp} \tag{3}$$

where

$$u = \frac{1}{\pi}\int_0^L \beta(\omega, z)\, dz \tag{4}$$

$f(u)$ represents $|\rho_i(\omega)|$ in the lossless case. z is substituted with $p = 2\pi(z/L - 1/2)$ and z/L is treated in terms of the electrical length [2]. Since the taper profiles such as the exponential or Chebyshev taper are previously given, the electrical length is calculated from (4) and then the frequency response corresponding to the electrical length of a dispersive tapered line is calculated from (2). If the desired response is established arbitrarily, it is difficult to calculate (2) and (3) because the taper profile is not known. This problem can be solved by the following generalized Taylor's procedure

TABLE I
Initial and Optimum Values of Null Points in the Lossless Case

	u_1	u_2	u_3	u_4	u_5
$u_{n,0}$	1	2	3	4	5
#1	0.83198	1.71024	2.72194	3.76768	4.83998
#2	1.23293	1.86008	2.61598	3.83528	5.10789

$$f(u) = \frac{1}{2}\left(\ln\frac{Z_2}{Z_1}\right)\frac{\sin \pi u}{\pi u}\,\frac{\prod_{n=1}^{N}\left(1-\left(\frac{u}{u_n}\right)^2\right)}{\prod_{n=1}^{N}\left(1-\left(\frac{u}{n}\right)^2\right)}, \quad n = 1, 2, \cdots, N. \tag{5}$$

This equation involves the frequency response of exponential taper in the case of $u_n = n$ and provides various responses according to the perturbation of u_n. Here N determines the passband. For example, $N = 5$ means 5.5 : 1 passband and the outer band of the passband consists with the frequency response of the exponential taper. When the number of the peak values of lobe-like frequency response arbitrarily established is N in the lossless case, the null point u_n must be properly chosen for desired response. This may be realized by an optimization process. Let $f_m(u)$ be m'th peak value and the error function $E(\boldsymbol{U})$ is defined as follows:

$$E(\boldsymbol{U}) = \sum_{m=1}^{N} |\ln(f_m(\boldsymbol{U}_l)/S_m(\omega))|^2, \qquad m = 1, 2, \cdots, N \tag{6}$$

in which l is the iteration number and $\boldsymbol{U} = [u_1\ u_2\ u_3 \cdots u_n]$. The least square method is used for minimization of the error function. Minimization of $E(\boldsymbol{U})$ is achieved by updating $\boldsymbol{U}$ to reduce the difference between the performance $f_m(\boldsymbol{U})$ achieved at any u_n and the specifications $S_m(\omega)$ representing an objective m'th peak values. Here $\boldsymbol{U}$ is updated as

$$\boldsymbol{U}_{l+1} = \boldsymbol{U}_l - \alpha_l \cdot \boldsymbol{H}^{-1}(\boldsymbol{U}_l) \cdot \nabla[E(\boldsymbol{U}_l)] \tag{7}$$

where $0 < \alpha_l < 1$ and $\boldsymbol{H}$ is the Hessian matrix. The difficulty of inverse Hessian matrix at each stage of iteration in (7) can be easily overcome by using an approximation to the inverse by the Davidon-Fletcher-Powell algorithm. For example, the optimum null points of the Chebyshev taper (#1) in which the tolerable reflection coefficient is 0.1 within the passband (5.5 : 1) and arbitrary taper (#2) in which the each lobe peaks are 0.02, 0.02, 0.05, 0.05, 0.02, successively, shown in Table I, respectively. Here $u_{n,0}$ is the initial values and consists with the null points of the exponential taper. Since the locus of reflection coefficients in the u domain cannot be distinguished from any phase constant, $u = \beta L/\pi$ may be used instead of (4) [4]. Therefore, although $\beta(\omega, z)$ has not been known, the u domain can be established by making values of βL varied within the involved range.

As a consequence, the viewpoint that the optimum null point can be determined for arbitrarily specified frequency response implies the possibility for synthesis in the lossy case. This is because if the frequency response synthesized in the lossless case varies by loss, the peak values vary. The peak values varied can be fitted to desired peak values by proper perturbation of null points.

For taking account of a loss, (1) is directly calculated by letting the characteristic impedance synthesized in the lossless case assume the initial value, hence representing the frequency response in the lossy case. The resulting peak values in the passband are subtracted from the desired values and the differences are added to the desired peak values. This makes the new objective peak values varied at each iteration to extract the optimum null points. The new objective peak values is expressible as

$$S_m^{k+1}(\omega) = D_m(\omega) - R_m^k(\omega) + S_m^k(\omega), \qquad m = 1, 2, \cdots, N \quad (8)$$

in which k is the iteration number and $S_m^{k+1}(\omega)$ is substituted into (6). This varying objective peak values consist of m'th peaks. $D_m(\omega)$ is the initial desired one and $R_m(\omega)$ means the m'th peaks calculated in (1). $S_m^0(\omega)$, namely the initial state of varying objective peaks, is substituted by $D_m(\omega)$. The summary of the algorithm mentioned above is as follows.

1) Determine the passband and the initial desired peak values $D_m(\omega)$ of the frequency response.
2) Calculate the optimum u_m from (5)–(7) in the u domain for the lossless case. And synthesize the $Z(\omega, z)$. In this process, set $S_m^0(\omega) = D_m(\omega)$ for the $k = 0$.
3) Calculate $|\rho_i(\omega)|$ in the f domain for the lossy case using (1) and $Z(\omega, z)$ previously synthesized and obtain the corresponding m'th peaks $R_m(\omega)$.
4) Obtain the objective peaks $S_m^{k+1}(\omega)$ varied at each iteration and repeat the above steps from 2) until

$$\sum_{m=1}^{N} |\ln(R_m^k(\omega)/D_m(\omega))|^2 \leq \epsilon, \qquad m = 1, 2, \cdots, N. \quad (9)$$

The covergence criteria adopted throughout are 10^{-10} for (6) and 10^{-8} for (9), respectively, and the least square method is used. If the criterion of (9) is satisfied, the corresponding $Z(\omega, z)$ provides the final taper profile for the lossy case.

III. Numerical Results and Verification

If a microstrip transformer for $Z_2/Z_1 = 2$ ($Z_1 = 50\ \Omega$ at $f = 0$) is to be designed for example, the frequency characteristics of the attenuation and phase costants, which are frequency and distance-dependent, have to be considered. The attenuation constant consists of α_c and α_d due to the loss of strip conductor and dielectric, respectively. In this paper, the well-known closed forms [7], [8] are used for the attenuation constant. The phase constant is expressible as $\beta(\omega, z) = \omega\sqrt{\epsilon_{r,\mathrm{eff}}(\omega, z)}/c$ in which the effective permittivity $\epsilon_{r,\mathrm{eff}}(\omega, z)$ [9] has dispersive characteristic and c is the velocity of light. In the optimization process, the varying taper profile is set by maintaining the strip width at $f = 0$ for each repetition in which the variation of characteristic impedance, according to the variation of frequency under the established taper, is also accounted. Fig. 2 represents the Chebyshev response optimized for the lossless case (solid line) having the tolerable reflection coefficient $\rho_l = 0.1$ in 5.5 : 1 passband and the aspects of variation by loss for arbitrary microstrip configurations (A and B). Here h, t, and ρ_c represent substrate height, strip thickness, and resistivity of strip conductor, respectively. Fig. 2 shows that the loss only affects the attenuation. If the frequency response in the lossy case is to be the Chebyshev response, the characteristic impedance of taper previously established under the lossless state has to be modified. For this modification, the synthesis has been achieved using (1)–(9) after six iterations for A and B shown in Fig. 2. Fig. 3 is the frequency response in the domain showing a shift of null points that are used for synthesis of taper profile in the lossy case and represents the variations of the electrical length. The optimized null points are shown in Table II.

This result is applied to (3) synthesizing the characteristic impedance $Z(\omega, z)$ and then the frequency response in the lossy case

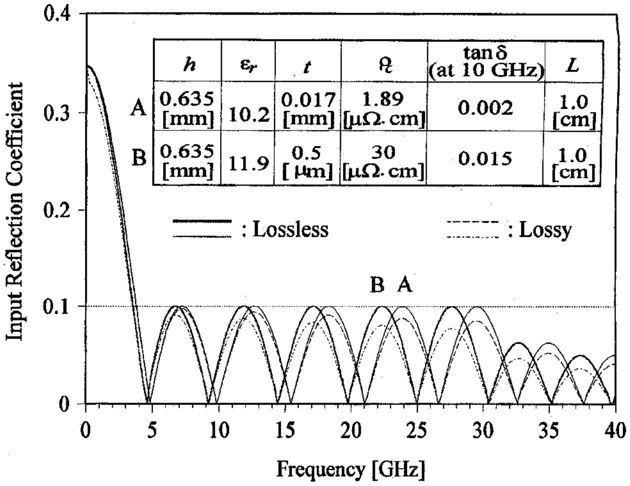

Fig. 2. Frequency characteristics due to variation of frequency in the lossy case (Chebyshev taper in the 5.5 : 1 passband).

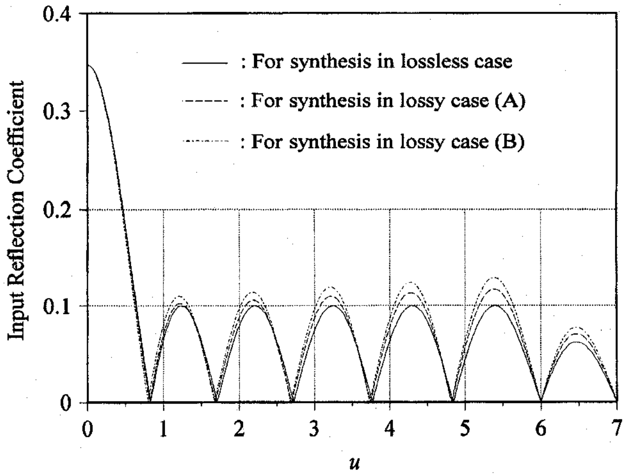

Fig. 3. Input reflection coefficients in the u domain for synthesis of characteristic impedance.

TABLE II
Initial and Optimum Values of Points in the Lossy Case

	u_1	u_2	u_3	u_4	u_5
u_n^0	0.83198	1.71024	2.72194	3.76768	4.83998
A	0.81789	1.68730	2.69453	3.74130	4.81959
B	0.79808	1.67474	2.68432	3.73334	4.81370

from (1) is shown in Fig. 4. This pattern represents the Chebyshev response in the lossy case. The microstrip taper profile in terms of characteristic impedance is shown in Fig. 5 where the lossless and lossy cases are compared. This figure represents maximum error of 3.604% and 7.528% for A and B, respectively, in each case. For verification, the synthesized taper was subsequently evaluated by modeling the nonuniform line with a large number of short, equal-length constant impedance segments whose values of impedance follow the synthesized impedance profiles shown in Fig. 5. Five hundred segments are reasonable for reducing an error due to shifts of null points by variations of phase constants. The evaluated responses are shown in Fig. 4. These plots show that the approximate equation (1) wrongly predicts a reflection of 0.3466 at $f = 0$ whereas the correct value is 0.3333, and that (1) for the reflection is accurate as

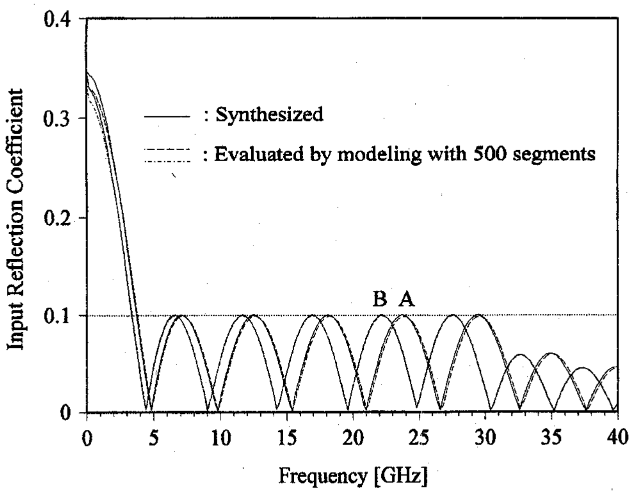

Fig. 4. Synthesized frequency characteristics in the lossy case.

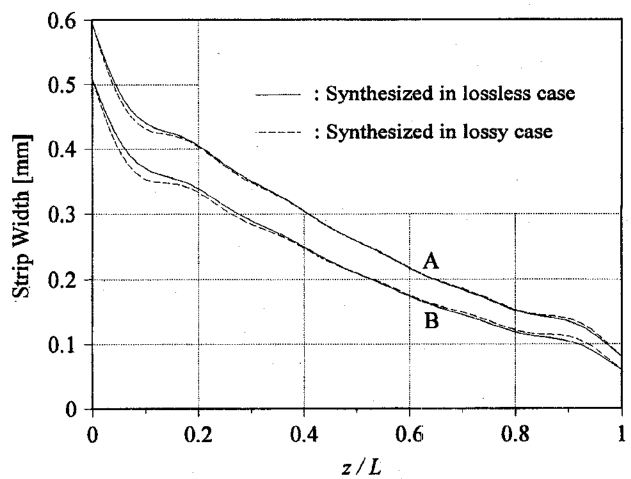

Fig. 5. Synthesized strip width in the lossy case ($f = 0$).

long as ρ_i is small. The exact numerical results in this paper show that the present theory provides a generalized theory for determining the impedance taper profile in lossy and dispersive media.

IV. Conclusion

A new efficient synthesis technique for the specified frequency response of lossy and dispersive tapered transmission line has been presented. This technique was accomplished by the optimization process to extract the optimum null points for the synthesis of the desired taper profile in the existence of a loss and dispersion. The results of synthesizing a microstrip transformer for example shows that the present synthesis technique with loss is important for design of high-frequency and high-density integrated circuits giving effect on the determination of the electrical length.

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Mode Orthogonality Relations and Field Structure in Chirowaveguides

E. O. Kamenetskii

***Abstract*—By analyzing the vector and scalar equations for chirowaveguides, two forms of mode orthogonality relations are obtained: the vector formulated orthogonality and the scalar formulated orthogonality. The first one is applicable to the general case of open chiroplasma or chiroferrite waveguides. It is shown that for two parallel-plate isotropic chirowaveguides, these two forms of orthogonality relations differ. Based on mode orthogonality relations, it is shown that in chirowaveguides the polarization of so-called complex modes differs from that of propagating or evanescent modes. The correlation between field components of two complex modes that transfer active power flow in chirowaveguides is obtained.**

I. Introduction

A number of problems related to chirowaveguides have been investigated and reported [1]–[13]. For example, dispersion characteristics and field distributions in parallel-plate [1]–[3], open-slab [4], [5], circular [2], [6], [7], and closed rectangular [8], [9] chirowaveguides have been studied. The surface waves in chiral layers have been analyzed noting elliptically polarized transverse electric and magnetic fields in the layers [10]. The theory of wave propagation in chiroplasma and chiroferrites [11] and the theory of chiroferrite waveguides [12], have also appeared. It has been pointed out in [13] that modes in chirowaveguides have interesting and useful properties of power orthogonality.

The power orthogonality (or vector formulated orthogonality relations) obtained in [13] for isotropic chirowaveguides may be easily extended to a more general case of lossless open chiroplasma or chiroferrite waveguides. Together with this type of orthogonality, one can also obtain the scalar formulated orthogonality relations. We will show that the scalar formulated orthogonality are not derived from the vector formulated orthogonality.

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